13 The Affleck-Dine-Seiberg Superpotential

13.1 Symmetry and Holomorphy

Consider $SU(N)$ SUSY QCD with $F < N$ flavors.

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
ϕ,Q	$\Box$	$\Box$	$\mathbf{1}$	1	$\frac{F-N}{F}$
$\overline{\phi}$,$\overline{Q}$	$\overline{\Box}$	$\mathbf{1}$	$\overline{\Box}$	-1	$\frac{F-N}{F}$

Recall that classically there is a (D-flat) moduli space

$$\langle\overline{\phi}^*\rangle = \langle\phi\rangle = \begin{pmatrix} v_1 & & \\ & \ddots & \\ & & v_F \\ 0 & \dots & 0 \\ \vdots & & \vdots \\ 0 & \dots & 0 \end{pmatrix} \tag{13.1}$$

At a generic point in the moduli space the $SU(N)$ gauge symmetry is broken to $SU(N-F)$. There are $2NF - F^2$ broken generators, so of the original $2NF$ chiral supermultiplets only F^2 singlets are left massless. We can describe these light degrees of freedom in a gauge invariant way by

$$M_i^j = \overline{\phi}^{j\alpha}\phi_{\alpha i} \tag{13.2}$$

Because of holomorphy the only renormalization of M is the product of wavefunction renormalizations for ϕ and $\overline{\phi}$.

The axial $U(1)_A$ symmetry which assigns each quark a charge 1 is broken by instantons. To keep track of selection rules we can define a spurious symmetry:

$$\begin{aligned} Q &\to e^{i\alpha}Q \\ \overline{Q} &\to e^{i\alpha}\overline{Q} \\ \theta_{\rm YM} &\to \theta_{\rm YM} + 2F\alpha \end{aligned} \tag{13.3}$$

so

$$\Lambda^b \to e^{i2F\alpha}\Lambda^b \tag{13.4}$$

To construct the effective superpotential we can make use of the following chiral superfields.

$$\begin{array}{ccc} & U(1)_A & U(1)_R \\ W^aW^a & 0 & 2 \\ \Lambda^b & 2F & 0 \\ \det M & 2F & 2(F-N) \end{array} \tag{13.5}$$

Note that $\det M$ is the only $SU(F)\times SU(F)$ invariant we can make out of M. A general term in the Wilsonian superpotential contains

$$\Lambda^{bn}(W^aW^a)^m(\det M)^p \tag{13.6}$$

The two symmetries require:

$$0 = n2F + p2F \tag{13.7}$$

$$2 = 2m + p2(F-N) \tag{13.8}$$

The solution is

$$n = -p = \frac{1-m}{N-F} \tag{13.9}$$

Since $b > 0$ we can only have a sensible weak coupling limit if $n \geq 0$, which implies $p \leq 0$ and $m \leq 1$. Since W^aW^a has derivatives, locality requires $m \geq 0$ and integer valued. So there are only two terms $m = 0$ and $m = 1$. The $m = 1$ term is just the tree-level field strength term (which is restricted by the periodicity of $\theta_{\rm YM}$ to be proportional to $b \ln \Lambda$). So we see that the gauge coupling receives no non-perturbative renormalizations. The other term $(m = 0)$ is:

$$W_{\rm ADS} = C_{N,F}\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}} . \tag{13.10}$$

13.2 Consistency of $W_{\rm ADS}$: Moduli Space

We can check whether this superpotential is consistent by constructing effective theories with fewer colors or flavors by going out in the classical moduli space or adding mass perturbations. Consider giving a large VEV, v, to one flavor. We can then construct an effective theory for $SU(N-1)$ with $F-1$ flavors. The running coupling in the low-energy theory

$$\frac{8\pi^2}{g_L^2(\mu)} = b_L \ln\left(\frac{\mu}{\Lambda_L}\right) \tag{13.11}$$

should match onto the running coupling of the high-energy theory

$$\frac{8\pi^2}{g^2(\mu)} = b \ln\left(\frac{\mu}{\Lambda}\right) \tag{13.12}$$

at the scale v:

$$\frac{8\pi^2}{g^2(\mu)} = \frac{8\pi^2}{g_L^2(\mu)} + c \tag{13.13}$$

where c represents threshold corrections, which vanish in the $\overline{\mathrm{DR}}$ scheme. So

$$\begin{aligned} \left(\frac{\Lambda}{v}\right)^b &= \left(\frac{\Lambda_L}{v}\right)^{b_L} \\ \frac{\Lambda^{3N-F}}{v^2} &= \Lambda_{N-1,F-1}^{3N-F-2} \end{aligned} \tag{13.14}$$

If we write the light $(F-1)^2$ degrees of freedom as $\widetilde{M}$ we have

$$\det M = v^2 \det \widetilde{M} + \ldots \tag{13.15}$$

Plugging these results into $W_{\mathrm{ADS}}(N,F)$ we reproduce $W_{\mathrm{ADS}}(N-1,F-1)$ if $C_{N,F}$ is only a function of $N-F$.

Giving equal VEVS to all flavors, $SU(N) \to SU(N-F)$ and we have:

$$\frac{\Lambda^{3N-F}}{v^{2F}} = \Lambda_{N-F,0}^{3(N-F)} \tag{13.16}$$

so

$$W_{\mathrm{eff}} = C_{N,F} \Lambda_{N-F,0}^3 \tag{13.17}$$

Which agrees with our holomorphy arguments for pure SUSY Yang-Mills.

13.3 Consistency of W_{ADS}: Mass Perturbations

Now consider giving a large mass, m to one flavor, and match on to the effective $SU(N)$ theory with $F-1$ flavors. Matching the coupling at the scale m gives:

$$\begin{aligned} \left(\frac{\Lambda}{m}\right)^b &= \left(\frac{\Lambda_L}{m}\right)^{b_L} \\ m\Lambda^{3N-F} &= \Lambda_{N,F-1}^{3N-F+1} \end{aligned} \tag{13.18}$$

Using holomorphy the superpotential is

$$W_{\text{exact}} = \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}} f(t) \tag{13.19}$$

where

$$t = mM_F^F \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{-1}{N-F}} \tag{13.20}$$

Taking the weak coupling limit $\Lambda \to 0$, $m \to 0$, with t arbitrary we find

$$f(t) = C_{N,F} + t \tag{13.21}$$

so

$$W_{\text{exact}} = C_{N,F} \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}} + mM_F^F \tag{13.22}$$

The equations of motion for M_i^F and M_F^j imply that M has the block diagonal form

$$M = \langle\phi\rangle = \begin{pmatrix} \widetilde{M} & 0 \\ 0 & M_F^F \end{pmatrix} \tag{13.23}$$

The eq. of motion for M_F^F gives

$$\frac{C_{N,F}}{N-F} \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}} = mM_F^F \tag{13.24}$$

Plugging in we find

$$W_{\text{exact}} = (N-F+1)\left(\frac{C_{N,F}}{N-F}\right)^{\frac{N-F}{N-F+1}} \left(\frac{\Lambda_{N,F-1}^{3N-F+1}}{\det\widetilde{M}}\right)^{\frac{1}{N-F+1}} \tag{13.25}$$

Thus we have a recursion relation:

$$C_{N,F-1} = (N-F+1)\left(\frac{C_{N,F}}{N-F}\right)^{\frac{N-F}{N-F+1}} \tag{13.26}$$

As we will see an instanton calculation is reliable for $F = N-1$, and this calculation determines $C_{N,N-1} = 1$ in the $\overline{\mathrm{DR}}$ scheme, and hence

$$C_{N,F} = N - F \tag{13.27}$$

We can also add masses for all the flavors. Holomorphy determines the superpotential to be

$$W_{\mathrm{exact}} = C_{N,F} \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{\frac{1}{N-F}} + m^i_j M^j_i \tag{13.28}$$

The equation of motion for M gives

$$\begin{aligned} M^j_i &= (m^{-1})^j_i \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{\frac{1}{N-F}} \\ &= (m^{-1})^j_i \left(\det m \Lambda^{3N-F} \right)^{\frac{1}{N}} \end{aligned} \tag{13.29}$$

Matching the gauge coupling at the mass thresholds gives

$$\Lambda^{3N-F} \det m = \Lambda^{3N}_{N,0} \tag{13.30}$$

So

$$W_{eff} = N \Lambda^3_{N,0} \tag{13.31}$$

Which agrees with our holomorphy arguments for pure SUSY Yang-Mills and determines the parameter $a = N$. Thus

$$\langle \lambda^a \lambda^a \rangle = -32\pi^2 \Lambda^3_{N,0} \tag{13.32}$$

Thus starting with $F = N-1$ we can consistently derive the correct ADS effective superpotential for $0 \le F < N-1$, and gaugino condensation for $F = 0$.

13.4 What Physics Generates W_{ADS}?

Recall that

$$W_{\mathrm{ADS}} \propto \Lambda^{\frac{b}{N-F}} \tag{13.33}$$

while instanton effects are suppressed by

$$e^{-S_{\rm inst}} = \Lambda^b \tag{13.34}$$

So for $F = N - 1$ is is possible that instantons can generate $W_{\rm ADS}$. Since $SU(N)$ can be completely broken in this case, we can do a reliable calculation. When all VEV's are equal the ADS superpotential predicts fermion masses of order

$$\frac{\delta^2 W_{ADS}}{\delta\phi_i \delta\overline{\phi}^j} = \frac{\Lambda^{2N+1}}{v^{2N}} \tag{13.35}$$

and a vacuum energy density of order

$$\left|\frac{\delta W_{ADS}}{\delta\phi_i}\right|^2 = \left|\frac{\Lambda^{2N+1}}{v^{2N-1}}\right|^2 \tag{13.36}$$

Looking at a single instanton vertex we find $2N$ gaugino legs (corresponding to $2N$ zero-modes) and $2F = 2N - 2$ quark legs. All the quark legs can be connected to gaugino legs by the insertion of a scalar VEV. The remaining two gaugino legs can be converted to two quark legs by the insertion of two more VEV's. Thus a fermion mass is generated. The dimensional analysis works because the only other scale in the problem is the instanton size ρ, and the integration over ρ is dominated by the region around

$$\rho^2 = \frac{b}{16\pi^2 v^2} \tag{13.37}$$

Forcing the quark legs to end at the same space time point generates the F component of M, and hence a vacuum energy of the right size. From our previous arguments we recall that we can derive the ADS superpotential for smaller values of F from the instanton case, so in particular we can derive gaugino condensation for zero flavors from this instanton calculation with $N - 1$ flavors.

For $F < N-1$ instantons cannot generate $W_{\rm ADS}$ since at a generic point in the classical moduli space with $\det M \neq 0$ the $SU(N)$ gauge group breaks to $SU(N-F) \supset SU(2)$. Matching the gauge coupling in the effective theory at a generic point in the classical moduli space gives

$$\Lambda^{3N-F} = \Lambda^{3N}_{N-F,0} \det M \tag{13.38}$$

In the far infrared the effective theory splits into and $SU(N - F)$ gauge theory and F^2 gauge singlets described by M. However these sectors can be coupled by irrelevant operators. Indeed they must be, since by themselves the $SU(N-F)$ gauginos have an anomalous R symmetry. The R symmetry of the underlying theory was spontaneously broken by VEV's but it should not be anomalous. An analogous situation occurs in QCD where a irrelevant operator (the Wess-Zumino term) is required to account for the anomalous decay $\pi^0 \to \gamma\gamma$. The correct term is in fact present since τ_{eff} depends on $\ln \det M$ through the matching condition (13.38). The relevant term is

$$\begin{aligned} &\int d^2\theta \ln \det M W^a W^a + h.c. \\ &= \left(Tr(F_M M^{-1}) \lambda^a \lambda^a + \text{Arg}(\det M) F^{a\mu\nu} \widetilde{F}^a_{\mu\nu} + \ldots \right) + h.c. \end{aligned} \tag{13.39}$$

The second term can be seen to arise through triangle diagrams involving the fermions in the massive gauge supermultiplets. Note $\text{Arg}(\det M)$ transforms in the correct manner to be the Goldstone boson of the spontaneously broken R symmetry. The eq. of motion for F_M gives

$$\begin{aligned} F_M &= M^{-1} \langle \lambda^a \lambda^a \rangle \\ &\propto M^{-1} \Lambda^3_{N-F,0} \\ &\propto M^{-1} \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{\frac{1}{N-F}} \end{aligned} \tag{13.40}$$

This gives a vacuum energy density that agrees with the ADS calculation. This potential energy implies that a non-trivial superpotential was generated for M, and since the only superpotential consistent with holomorphy and symmetry is W_{ADS} we can conclude that for $F < N-1$ gaugino condensation generates W_{ADS}.

13.5 Vacuum Structure

Now that we believe W_{ADS} is correct what does it tells us about the vacuum structure of the theory? It is easy to see that $V = |F_i|^2$ is minimized as $\det M \to \infty$, so there is a "run-away vacuum", or more strictly speaking no vacuum. A loop-hole in this argument would seem to be that we have not included wavefunction renormalization effects, which could produce wiggles or even local minima in the potential, but it could not produce new vacua unless the renormalization factors were singular. It is usually assumed that this cannot happen unless there are particles that become massless at some point in the field space, which would also produce a singularity in the superpotential. This is just what happens at $\det M = 0$, where massive gauge supermultiplets. So we do not yet understand the theory without VEV's.

References

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